NBF-003-001205 Seat No. ________

B. Sc. (Sem. II) (CBCS) Examination

April / May - 2017

Mathematics : BSMT - 201 (A)

(Geometry, Trigonometry & Matrix Algebra)

Faculty Code : 003
Subject Code : 001205

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicate full marks of the question.

1 Give answers of all following questions : **20**

(1) Define singular matrix.

(2) For a square matrix A determine the type of matrix $A + A^T$.

(3) State Cayley-Hamilton theorem.

(4) If λ is an eigen value of A then what is the corresponding eigen value of A^5 ?

(5) What is the rank of $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(6) Verify whether the sequence $\left\{\frac{1}{n}\right\}_{n\geq 1}$ is convergent or not.

(7) Define Cauchy sequence.

(8) State general principle of convergence of sequence.

(9) Write the equation of right circular cylinder with radius r whose axis is parallel to $Y - axis$.

(10) What is the radius of right circular cylinder whose guiding curve is $x^2 + y^2 + z^2 = 4; x + y + z = 3$?

(11) $(\cos\theta + i\sin\theta)^5 (\cos 2\theta - i\sin 2\theta)^3$ = ________

(12) If 1, w are two roots of cube roots of unity then the third root is __________.

(13) If $x=\cos\theta+i\sin\theta$ *then* $x+\frac{1}{x}$ *is* __________.

(14) Write polar form of $1+\sqrt{3i}$.

(15) Find roots of $x^4-1=0$.

(16) Express tanh x in terms of exponential function.

(17) $\cosh^{-1}x=$__________

(18) $\cosh^2 x-\sinh^2 x=$__________.

(19) Find the value of $\log(-7)$.

(20) Find the real part of sin z.

2 (A) Attempt any **three** : **6**

(1) Define symmetric matrix and provide an example of the type.

(2) Discuss the convergence of a sequence $\left\{\sqrt{n+1}-\sqrt{n}\right\}_{n\geq 1}$.

(3) Find eigen values of $\begin{bmatrix} 2 & 5 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

(4) Explain homogeneous system of linear cautions with suitable example.

(5) Define : *(i)* cylinder, *(ii)* right circular cylinder.

(6) Prove that every convergent sequence is bounded.

(B) Attempt any **three** : **9**

(1) Find rank of $\begin{bmatrix} 2 & 5 & 1 & -1 \\ 3 & 6 & 1 & -2 \\ 4 & 7 & 1 & -3 \end{bmatrix}$

(2) Prove that eigen values of Skew-Hermitian matrix are either zero or purely imaginary.

(3) Discuss the consistancy of system of simulteneous linear equations.

(4) Find equation of a cylinder whose generator parallel to $\frac{x}{3}=\frac{y}{2}=\frac{z}{1}$ and passing through a guiding curve $x^2+2xy+y^2=1; z=0$.

(5) Prove that every convergent sequence has unique limit.

(6) Find eigen vectors of $\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(C) Attempt any **two** : **10**

(1) Derive equation of right circular cylinder with axis $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ and radius r.

(2) Using Cayley-Hamilton theorem, find inverse of $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(3) Solve the system of linear equations : $x-y+3z=4, 3x-4y+z=2, x-3y+z=-2$.

(4) Prove that each eigen value of a matrix A is non zero if and only if A is invertible.

(5) Prove that if $\lim\limits_{n\to\infty} a_n = l$ then

$$\lim_{n\to\infty}\left(\frac{a_1+a_2+a_3+\ldots+a_n}{n}\right)=l$$

3 (A) Attempt any **three** : **6**

(1) Simplify : $\frac{(\cos 3\theta+i\sin 3\theta)^4(\cos 4\theta-i\sin 4\theta)^5}{(\cos 4\theta+i\sin 4\theta)^3(\cos 5\theta+i\sin 5\theta)^{-4}}$.

(2) Prove that $\tanh^{-1}x=\sinh^{-1}\frac{x}{\sqrt{1-x^2}}$.

(3) Find real and imaginary part of $(1+i)^i$.

(4) If $z = \cos\theta + i\sin\theta$ then prove that $\frac{1+z^2}{1-z^2} = i\cot\theta$.

(5) If $\tan\frac{x}{2} = \tanh\frac{u}{2}$, prove that $\sinh u = \tan x$.

(6) Prove that $\sec h^2 x + \tanh^2 x = 1$.

(B) Attempt any **three** : **9**

(1) Solve : $7\sinh x + 20\cosh x = 24$.

(2) If $x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$, show that

$\lim_{n\to\infty} x_1 . x_2 \ldots x_n = -1$.

(3) Find roots of $x^4 + 1 = 0$.

(4) Prove that

$$\cos^8\theta = \frac{1}{128}\left[\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35\right].$$

(5) If $a = \cos\theta + i\sin\theta$ and $b = \cos\phi + i\sin\phi$, then prove

that $\cos(\theta + \phi) = \frac{1}{2}\left(ab + \frac{1}{ab}\right)$.

(6) Prove that $\tan\left[i\log\left(\frac{a-ib}{a+ib}\right)\right] = \frac{2ab}{a^2+b^2}$.

(C) Attempt any **two** : **10**

(1) State and prove De'Movire's theorem.

(2) Expand $\sin n\theta$ in terms of $\sin\theta$ *and* $\cos\theta$.

(3) Find real and imaginary parts of

$\sin^{-1}(\cos\theta + i\sin\theta), 0 < \theta < \frac{\pi}{2}$.

(4) If $a + ib = i^{i^{\cdots}}$, prove that $a = \frac{2}{\pi}\tan^{-1}\left(\frac{b}{a}\right)$ and

$b = -\frac{1}{\pi}\log\left(a^2 + b^2\right)$.

(5) Prove that

$$(a+ib)^{\frac{m}{n}} + (a-ib)^{\frac{m}{n}} = 2\left(a^2+b^2\right)^{\frac{m}{2n}}\cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right).$$